Vinculum

MATHEMATICAL ASSOCIATION OF VICTORIA

I had a little spreadsheet, nothing would it show,
But a flashing cursor, pointing where to go.
I type a little number, click a little plus—
And every number after that, adds without a fuss.

Poor old slide rule, have you any use?
Yes, sir, two sir, I'm no goose!
I'm good for history, slide me and look.
And also I'm very good to rule up your book.

(From *Millenial Nursery Rhymes*)

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FURTHERING KNOWLEDGE AND LINGUISTIC COMPETENCE: Learning with Kernel Ideas & Journals

Peter Gallin
URS RUF – University of Zurich (Switzerland)

Children who want to learn care little for traditional subject barriers. From the beginning, their curiosity and thirst for knowledge are directed towards the whole, though it is impossible to predict exactly what this 'whole' will be. Once the child has latched onto it, however, it becomes a personal '**kernel idea**' and begins to take effect. We can watch this happening with children every day.

> My seven year-old son Jonas holds a pocket diary with a miniature map under my nose. 'Daddy', he demands, 'tell me all about the world.' 'That's only a map of Switzerland', I reply. 'What? Only a map of Switzerland?' Jonas is disappointed. Flicking through the diary he had discovered a map, which aroused in him an interest in geography. But he wants to know about the whole thing: he wants to have the world explained, not just Switzerland. I try to get him interested in the map by mentioning a few names he knows from trips we have made. 'Look, here's Lake Geneva. And here's the Gotthard. And down there is Magliaso on Lake Lugano.' But my efforts are in vain. However much these names might fire his imagination at any other time, today they fall flat. What is the point of it all? Jonas does not want the details, he wants the whole — the world. I go down to the cellar and, from amongst the junk, unearth a rather battered-looking globe which one of my older children had been given years before. Jonas takes the globe and is serenely happy. Now he has what he wants: America, Asia, Africa, the oceans, the Mississippi, Greenland, the North Pole, Australia and the South Sea islands he has heard about in a children's story.

This little episode shows how many fortunate circumstances are needed before human curiosity achieves its object (The scene is quoted and translated from Peter Gallin / Urs Ruf: *Sprache und Mathematik in der Schule. Auf eigenen Wegen zur Fachkompetenz.* Illustriert mit sechzehn Szenen aus der Biographie von Lernenden. LCH-Verlag, Zürich 1990, distributed via Lehrmittelverlag Zürich.) It is thanks to several coincidences that Jonas gets hold of the globe at the right time. First his curiosity must be awakened and articulated in a manner which is understood. What takes shape in this subtle process we term **kernel idea**. In the request 'Daddy, tell me all about the world' the whole which Jonas has in mind is indicated vaguely. He has an idea of the goal and of the way in which he can reach it. His demand casts his father in the role of teacher and is accepted. But the father, too, is led by a kernel idea. He thinks Jonas has made a mistake and does not want to know about the world, but just about Switzerland. In the awkward dialogue between 'teacher' and 'student', the two different kernel ideas clash. It is the conflict between two different views of what comprises the object of learning, the 'whole'. Fortunately, Jonas does not let himself be led away from his original idea by his father's well-meant educational efforts. Thanks to the obstinacy of the 'student' and to the restraint on the part of the 'teacher', the two of them accidentally come up with the globe, just the educational medium which Jonas needs at that moment.

The climate of this little scene with Jonas is characteristic of independent learning. Two features here are of vital importance for learning in school.

- The learner aims for the whole right from the beginning and does not want to waste time with trivial details.
- The educator is first and foremost the patient listener, willing and able to revise his or her didactical concept and to be shown by the learner where and how supportive intervention is necessary.

Teaching which proceeds from kernel ideas appropriate both to the learners and the subject-matter challenges teachers and students alike to rethink and redefine their roles. Both sides are confronted with new tasks. The teacher is required to formulate kernel ideas which give the beginner something of a panoramic view of the subject or topic. Initial resistance to this view—which is above all a personal choice of the teacher's—should not be misinterpreted as an attack, but taken up as a challenge to creatively modify or redesign the kernel idea. In absorbing the germ of the learners' kernel idea and trying to approach the subject-matter from the perhaps quite foreign point of view of the student, a teacher opens up opportunities for the learners to tread their own paths of learning. Not only that: the teacher will frequently find that this method leads on to new, unexpected interpretations of old, familiar material. This demands not only patience and restraint, however, but also the ability to translate singular viewpoints and ideas, expressed in the often rudimentary language of the learners, into issues which will be fruitful for the subject at hand (i.e. relativistic transformation).

A productive dialogue about kernel ideas can only ensue if the learners are seriously involved. They too are in possession of kernel ideas about the subject-matter, although they are often not aware of the fact. Kernel ideas are not the product of a particular effort. They occur spontaneously whenever a person comes upon matter. As humans, we cannot avoid being in some sort of relationship with what we encounter. But to make this into a topic for study is demanding and requires courage. The relationship learners have with the subject-matter or with the act of learning is often so foreign that they do not want to talk about it and do not consider it relevant to teaching. 'I don't like Goethe' or 'Trigonometry is too hard for me'—who would willingly lay themselves bare by revealing these kernel ideas? Yet where should the learning process begin if not at the point the learners are really at? But their position is none other than that marked by their kernel idea. If the topics are to get through to the learners, to permeate and change them, each of them must first consciously take up his and her position in relation to the subject-matter and the teacher, and enter the dialogue from there. The learners must not simply accept the learning arrangement as it stands, but should take an active part in the search for kernel ideas which do justice both to themselves as learners and to the subject they are learning. Only when the subject-matter, in the course of a discussion of conflicting kernel ideas, has taken on a shape which expresses the whole in a way which is understandable and motivating for all the learners, is the beginning part—we call it the initialisation—of individual learning processes completed. Only then the process of independent and meaningful competence acquisition can begin.

How Can This Be Achieved With Twenty-Five Students?

A development project of the Zürich Department of Education gave us the opportunity of testing our concept 'individual paths to learning' over two years (1988-90) with classes in all school grades. Besides kernel ideas, an important feature in our work was the 'journal'. We have been using journal-writing for more than 20 years with High School classes in German (mother tongue) and Mathematics. Since 1980 we have been presenting this medium in courses and publications (*Tages-Anzeiger Zürich, 23. 9. 1980; Gymnasium Helveticum,* 1/82; *Praxis Deutsch,* Vol. 70, 1985; *mathematik lehren,* Vol. 9, 1985). In 1986 we circulated a collection of detailed descriptions of individual learning processes as demonstrated by journals, entitled *Lernen auf eigenen Wegen*. This manuscript formed the basis for our development project in the canton of Zürich described in this paper.

Both concepts try to perceive people and materials in their entirety. Whilst kernel ideas represent fixed points for orientation and stimulate individual learning processes, *journals* record the tracks made by the individuals in a wide-ranging subject area. The use of written language played a major role in the project. The act of writing enables thoughts and feelings to be slowed down and clarified, to take shape and elicit a response. In writing one assumes, in a particular way, responsibility for one's position and lays oneself open to criticism. Individualisation is conceivable without the development of higher written language competences, because the learners are allowed to use the language available to them at the time independently. If learners document their learning route like the traces of Hansel and Gretel, they are not wandering aimlessly in new territory, but become more familiar with the material with each independent excursion and gradually gain an overall view. If the students write, the teacher also gains time for counselling. And the tracks documented in the journals enable the teacher to recognise relatively easily how far each learner has progressed and what help they need at any particular moment.

The central concern of the concept 'individual paths to learning' is to guide the learners towards accumulation of regular knowledge and skills, without alienating them from their singular basis. This leads to a completely new evaluation of what the students produce and to a dramatically more comprehensive interpretation of achievement. Although it was not only the students but also the teachers who went their own ways in the project, certain constants emerged, which allow the wide variety of manifestations to be interpreted and the different activities to be judged. The following model, with four fields of learning and work is a rough guide and must not be misunderstood as, say, a linear teaching program which must always progress woodenly from point 1 to point 4.

At the beginning of any learning process are kernel ideas which capture in vague outlines the subject as a whole and provide an attractive invitation to subject-related action. Dealing with kernel ideas takes place in the journal at the individual level determined by each learner. It serves to build regular knowledge and skills. The overall task heading is 'document your route!'. In the preview, each learner explores the subject from his or her own point of departure. The goal or 'destination' of the route is an overall view of the subject-matter achieved in the final (retrospective) phase and a possession of the appropriate, independently developed algorithms.

Irrespective of grade and of the learners' individual cognitive potential, the students in the project were required to achieve in particular in the four areas defined as *preview, pathfinding, product* and *retrospective*. The following questions proved useful as a rough guide to independent learning and characterised the work of the learners in all subjects. They aim at both subject and linguistic competence applicable to all subject areas and are engendered by higher order educational objectives whose influence is normally confined to the lip-service paid them in the preambles to printed curricula.

1. **Preview**
   How does this subject-matter affect me?
   How and where does the teacher's kernel idea move me to action?

Which kernel idea do I allow myself to be led by?
Is there something which gives me personal access to the subject area?
What scope does the teacher's set task give me?

2. **Pathfinding**

How do I behave in problem solving?
Can I organise my work with the help of the journal?
Where do I stand and where am I stuck?
How and where does written language help me clarify my thoughts?
Do I dare document where I went wrong and can I learn from my mistakes?
Can I watch myself work and develop useful strategies?

3. **Product**

Can I assert myself with my knowledge and ability?
Do I know what is being demanded? Can I, do I want to measure up to this?
Can I recount what I have grasped in a way which is understandable?
When I am working, can I harmonise my own intentions with what is required of me from outside?
Could I present my achievements to a wider public?
Am I clear about my concept and do I have the necessary means?

4. **Retrospective**

What have I achieved?
Am I satisfied with my work and is it recognised?
What has changed? What are the consequences?
How do others act in comparable situations?
What can I learn from other students, experts or artists?

These questions are in line with four registers of independent work which a responsible person has at his or her disposal. Teachers who take first steps in independent learning with their students will first concentrate on just one of these questions, modifying it to suit the age-group and limiting the scope as necessary. As proficiency increases, several questions can be dealt with at the same time, and the time-span for independent learning approaches stretched from double lessons to half an academic year. Each register opens up fields with specific tasks, methods and assessment criteria. This can be seen in the table below.

Notes on the table

1. **Initialising assessment in the preview**

At the beginning of a learning phase the task is to establish the singular terms of the learner who is invited to respond to a kernel idea of the teacher's. The sole aim is to find out where and how the learners see themselves

Register of independence	Learner's task	Teacher's task	Assessment criterion	Assessment function
Preview Discovering a kernel idea of one's own motivation and formulating an individual task	Singular response to the teacher's kernel idea	Revelation of individual intentions and competences by means of relativistic transformation	Criteria which are generated in relation to the learner's singular system	To initialise a basis for assessment appropriate to the student and the subject-matter (initiative)
Path Learning to organise and steer the learning process independently	Documentation of personal route to learning in the journal	Feedback, response, recommendation, objection, encouragement, counselling	Criteria which are generated in relation to a developing system of concepts and problem-solving strategies (double aspect singular/regular)	To optimise individual learning behaviour and to expand competence (formative)
Product Evidence of current achievement capability	Synthesis of singular intentions and regular requirements	Ascertainment of difference between product and goal of task	Criteria which are generated in relation to a system of norms	To establish the individual level of achievement on a given scale (summative)
Retrospective Self-assessment (process and product)	Divergent comparison of own and other work (class and society)	Confirmation or correction of learner's self-assessment	General criteria which are generated in relation to the learning objectives and levels of school achievement	To take stock by comparison of different achievements and to plan the next learning phase or progression (diagnostic and prognostic)

moved to action by the subject-matter. The teacher's role is to reveal the individual nature and intention of this action, which holds the key to future achievement. Only when the learners, with the help of the teacher, succeed in developing their own kernel ideas within the framework of the subject-matter — delineated for them by the teacher's kernel idea — does their action gain a direction and an objective (preview). We are dealing here with the heart of personality development through schooling which — if it is to have credibility — must prove itself in a dialogue with school subjects. Only when the learners are themselves convinced that bothering about language, mathematics and other subjects is worthwhile do they open up to the opportunity for independent learning. Personal judgement, bound up as it is with the will to confront the subject-matter school offers, is the prerequisite if the learners are to be receptive of criticism and advice and use these productively. The end of this phase is a task set for the individual to be completed within a given time and with specific goals which play a substantial role in subsequent assessment and evaluation of the students' achievement (initiative achievement assessment).

2. **Reflection and direction of the individual learning process**

   In the journal learners document their individual 'path' of learning. They make themselves accountable for their behaviour and their insights during problem-solving. As they do not yet have the regular terms and procedures at their disposal, their achievement cannot be judged in relation to the norm, but only in relation to their singular capacity for achievement. By means of *feedback* appropriate both to the subject-matter and the student, the teacher confronts the student with an individual standard and shows the student where he or she stands and in which direction he or she could develop. In counselling the student, the teacher is hardly likely to be able to separate assessment and evaluation. Quality judgements of the student's work will be practically inevitable in the teacher's feedback. These will first refer exclusively to the learner's singular standard, but will gradually increase the reference to regular norms. This complementary use of singular and regular assessment criteria enables the learners increasingly to judge for themselves where their strengths and weaknesses lie and how they can make fruitful use of this knowledge in future work (formative achievement assessment).

3. **Ascertaining current achievement levels**

   The end of a learning process in a subject area is heralded when the learner can express his or her *singular* intentions in *regular* forms. Creative work in an area in which the learner feels at home results in a synthesis of the singular and the regular. The learners succeed in delineating and solving appropriate problems within a larger, complex structure. They are now in a position to penetrate subject-matter themselves and design the appropriate products which can be assessed according to regular standards and which indicate the current level of the learner's achievement capability (summative achievement assessment).

4. **Results and planning**

   The learning process is characterised by a gradual progression towards regular concepts and algorithms. It is, however, a vital requirement that the singular is not smothered or suppressed but encouraged to develop in its own way. The diverging comparison with the achievement and behaviour of others enables the learner to review critically his or her own actions retrospectively and to measure them against external norms. Two-dimensional overall assessment draws on both person-related value judgements concerning the learner's behaviour in the journal (learning process assessment), and norm-related value judgements based on products (learning product assessment). The project showed that some learners prove to be 'path specialists' whilst others tend to be 'product specialists'. Their achievements can be weighted in such a way that the strengths in one area outweigh the weaknesses in another. Assessment which draws as well on path as on product and which makes room for well-founded self-assessment on the part of the learners not only has a formative and summative function, but also creates a basis for talks with parents, authorities or other teachers when decisions about selection processes or the student's subsequent school careers are to be made (diagnostic and prognostic achievement assessment).

Differentiated achievement requirements and a clear distinction between process and product assessment (two-dimensional achievement assessment) led in all project classes to an increase in the students' will to achieve. The commitment to documenting their own learning process in the journals prevented high-achieving students from resting on their laurels, whilst weaker students were able to perform well or very well under their more limited abilities. Let us now examine an example in more detail.

Excerpts From Astrid's And Ueli's Journals

Astrid and Ueli are in the first grade of High School (year 7), in a class which took part in our project. We assist in a lesson in Mathematics: the order of rational numbers. The kernel idea is presented in the shape of a veritable bicycle which has 18 gears and belongs to Luki. That special and long bicycle has no saddle but a chair and is driven by a lying cyclist. Many of the students have already ridden this amazing vehicle and the question how to change the gears in order from the first to the last is thus a most topical concern. Technically speaking, the students need to establish the hierarchy of the gears. After a certain amount of puzzling the mechanics are examined in more detail. By the pedals there are three chainweels with 52, 38 and 26 sprockets respectively. On the rear wheel there are six chainwheels with 32, 26, 21, 18, 16 and 14 sprockets

respectively. In their first phase of exploration, the children discover that the fractions formed by the number of sprockets front and back are a measure of the gear magnitude. The gear 'front 52—rear 21', for example, can thus be described as 52/21. This means that when the pedals complete a revolution with their 52-sprocket chainweel, the rear wheel completes 52/21 revolutions with its 21-sprocket wheel. How can this information be used to determine the hierarchy of the 18 gears? The following excerpt from a journal documents the 'journey' made by one pupil we have called Astrid. She has her difficulties with mathematics and does not arrive at the goal. Nevertheless, she has the chance to produce a good piece of work and her interest in the topic remains keen.

1.9.89 LUKI'S BIKE

First of all I'll try to find a common denominator for the numbers.

- First I'll try multiplying 32·26. Then I'll see if the other denominators are in 832.

denominator \ common denominator	32·26=832	26·32·21·18·16·14 = 70'447'104	18·16·14 = 4032
26	✓	✓	
32	✓	✓	✓
21		✓	✓
18		✓	✓
16	✓	✓	✓
14		✓	✓

I don't know exactly what system I should use. I could multiply all the numbers together. Perhaps that would work. But then I wouldn't have the smallest denominator. ✓

- What about the 3 smallest numbers? ✓
  No luck. 21^{26} doesn't work.
- Let's try 26·18·16·14.
  That's OK. But the number is too high for a denominator. ✓
- I'll try with a mixture of the numbers: 16·21·32 = 10752.
  That can't be divided by 26 and 18. ✓
- All right. I'll try 18·21·32 = 12096. Doesn't work either.
- I've tried other numbers (different combinations with the denominators), but it's really difficult to come up with something this way. I'll just have to take 84 672 as the common denominator. It's bound not to be the smallest, but at least I'll get an answer, even if it's not the right one. ✓

denominator \ common denominator	84672 14·16·18·21 =	16·21·32 = 10752	18·21·32 = 12096
32	✓	✓	✓
26			
21	✓	✓	✓
18	✓		✓
16	✓	✓	✓
14	✓	✓	✓

1st gear 26/32 =
Oh no! I've made a mistake. 84 672 can't be divided by 26. I think I'm going to give up. I don't want to do the whole thing again with 70 447 104. Let's see what happens tomorrow. ✓✓

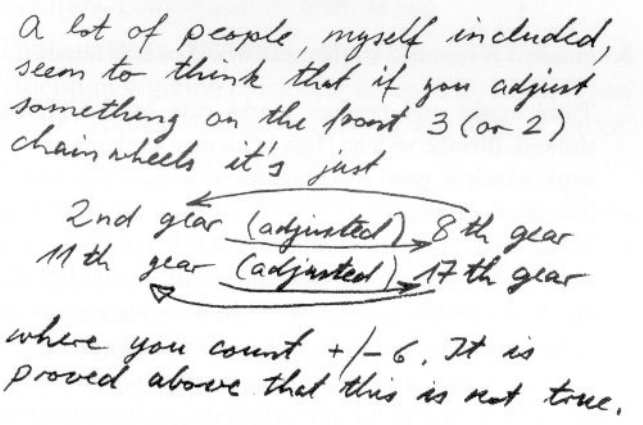
Astrid grasps what the task is about, but does not reach the required goal. She does not manage to transform the 18 fractions to a common denominator. Along with most of her classmates, she is not yet familiar with the standard procedure for such an exercise. Nonetheless, she sets about finding a solution and faithfully documents each step. Astrid's individual 'journey' exemplifies several important characteristics of our project.

- Astrid is making use of written language, an opportunity which our schools seldom afford. Yet things can be seen more clearly if we write them down (Heuristic function of language).
- Astrid is thinking about her own way of solving the problem and can later improve on the procedure she has chosen by comparing it with others (Metacognition).
- Astrid assumes responsibility for the course her work takes and for the point at which she breaks off (Responsibility).
- Astrid is eager to find out what comes next. Her personal confrontation with the subject-matter in the privacy of the journal is naturally complemented by the 'publicity' of the classroom and creates a basis for learning within a community (Divergent comparison).
- The journal text shows where Astrid stands. She senses what the magnitude of the common denominator could be, but lacks faith in her own powers of thought. She breaks off a promising attempt, although the common denominator she has found, 26¥18¥16¥14, is only four times larger than the smallest (Securing singular evidence).
- The teacher realises at once where Astrid's difficulties lie and can write a few brief remarks to encourage Astrid to continuer on her own (Formative assessment).

Astrid has made a genuine attempt at solving the problem within her capabilities. Her good work at the level of problem-solving behaviour can only be recognised because she has documented so meticulously the path she chose. Although she did not attain the set goal, her ability to achieve at the level of higher-order educational goals irrespective of subject has been proved. We will take account of this when calculating Astrid's report mark in mathematics. As Astrid has dealt with the problem in a way which suits her, she is not frustrated when she breaks off her work, but keen to hear what her classmates have come up with.

Journal work is not only encouragement for weaker students, however. It also spurs higher achievers to give of their best. Ueli, for example, soon finished with the fractions and penetrated deeper into the subject-matter. The extraordinary comment he adds to his long list of correctly

ordered gears testifies to this. In solving the problem Ueli has overcome a preconception and made an interesting discovery, but the tortuous language does not do full justice to the thought it expresses. He does not yet possess the regular language competence to articulate adequately what he has grasped, an insight which is after all so much more complex than Astrid's reflections. For the teacher it is a difficult, but imperative and satisfying task to recognise this imbalance and use it to fuel success in learning. The transformation of Ueli's text into regular language shows up not only his brilliant mathematical achievement, but also his linguistic deficiency. Ueli's satisfying experience of being understood in his peripheral reflections on a mathematical task reinforces his confidence in his independent thinking and, moreover, encourages him to write down any other discoveries he makes, perhaps in a form which is more accessible to the reader.



Ueli has discovered that for our 18-gear bicycle with its six chainwheels we cannot count the numbers with two-digits in base six. If we want to go up through the gears in the correct order, we cannot simply treat the forward chainwheels as superordinate and change them only after we have changed through from the largest to the smallest chainwheel at the rear. The two-digit series 00, 01, 02, 03, 04, 05, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25 does not reflect the hierarchy of the 18 gears. (In this system the first digits 0, 1 and 2 correspond to the smallest, middle and largest front chainwheel. The second digits 0, 1, 2, 3, 4 and 5 correspond to the rear chainwheels from the largest to the smallest.) In this abstract system of notation, the correct hierarchy of the gears on Luki's bicycle would be 00, 01, 10, 02, 03, 11, 04, 20, 12, 05, 21, 13, 14, 22, 15, 23, 24, 25.

Can Individualised Teaching Be Reconciled With Mark-Giving?

The 'individual paths to learning' project, from which the example of Astrid and Ueli is taken, sets out to show opportunities for forms of language, mathematics and basic science teaching which integrate and individualise and which are feasible within existing school structures. These include not only age grades, timetables, traditional subject divisions and school buildings, but also selection and achievement assessment through marking. These are conditions which particularly influence teaching at High School, the six-year academic secondary school in Switzerland. Thus any efforts to promote independent learning have no chance of success if they cannot in some way be incorporated into the traditional system of achievement evaluation. The following model of a non-linear, two-dimensional scheme of assessment can be harmonised with conventional marking procedure, yet also has considerable reforming potential. If a reform of the marking system is to have more than merely superficial effect, it must be confidently supported by teachers with positive experience of the new system under their belts, who started on familiar ground, gradually extended their concept of achievement and discovered new possibilities of teaching and learning.

The excerpts from Astrid's and Ueli's journals are a sound basis for assessment which recognises achievement in the individual's process of learning and takes account of behaviour in problem-solving. Astrid's work is spontaneously felt to be 'good', although it is not directly comparable with Ueli's work. If Ueli had only presented the well ordered list of all the fractions in his journal, without expressing his understanding, his achievement would have been satisfactory, but no more. What he expresses in his singular language, however, awkward as it may still be, bears witness to an excellent line of thought which would hardly have been noticed had he not been forced to verbalise his reflections. Value judgements such as these can be made fairly easily if only a rough classification scheme is adopted. In our project, '**good**', '**task completed**' and '**task not completed**' proved practicable verdicts on achievement documented in the journals, symbolised by two ticks, one tick or one tick crossed over respectively. The verdict '**lucky strike**' (three ticks) was reserved for surprising, original, unusual achievement. In this case, a fruitful mistake or a transparently traceable odyssey without a result can also count as a 'lucky strike', but not a solution following a conventional pattern. In our examples, Astrid is awarded two, Ueli three ticks.

task not completed	task completed	good	lucky strike
✓̸	✓	✓✓	✓✓✓

In writing a journal, Astrid has a fair chance of outdoing Ueli, whilst in conventional examinations, which we class as products, Ueli is the stronger. In our two-dimensional model of achievement assessment, Astrid and Ueli can move within two different but equal systems of values: **path-oriented** generation of knowledge and **goal-oriented** generation of products. Independent achievement is possible in both systems. By using a special form of **non-linear** marking, calculating marks on the basis of as well conventional examinations (product) as journals (path), we gave the students the chance to demonstrate their abilities relevant to the subject at hand **in those areas where their**

Report mark calculation table		Path: Mean number of tricks						
		0	0.5	1	1.5	2	2.5	3
Product: Mean mark of exams	1	1.0	2.0	2.5	3.0	4.0	4.5	5.0
	1.5	1.5	2.0	2.5	3.5	4.0	4.5	5.0
	2	1.5	2.0	3.0	3.5	4.0	4.5	5.0
	2.5	2.0	2.5	3.0	3.5	4.0	4.5	5.5
	3	2.0	2.5	3.5	4.0	4.5	5.0	5.5
	3.5	2.5	3.0	3.5	4.0	4.5	5.0	5.5
	4	3.0	3.5	4.0	4.0	4.5	5.0	5.5
	4.5	3.5	3.5	4.0	4.5	5.0	5.0	5.5
	5	4.0	4.0	4.5	4.5	5.0	5.5	6.0
	5.5	4.5	4.5	5.0	5.0	5.5	5.5	6.0
	6	5.0	5.5	5.5	5.5	6.0	6.0	6.0

strengths lie. Learners who can make optimum use of the journal as a 'workshop of mental activity' can be said to be making encouraging progress, even if they are dealing with simpler problems than their classmates. For this reason we weighted good journal results favourably against poor exam performance. The reverse was applied in the case of students who performed well in exams but had trouble with their journals. Good exam results weighed more than temporary problems with individual and independent problem-solving. This non-linear, two-dimensional approach to achievement assessment led not only to an almost complete eradication of examination stress and thus to more purposeful behaviour when working under pressure, but also to a significant increase in voluntary participation and interest in the subject.

The following table shows how journal achievement and product achievement were counterbalanced and a mark calculated for the students reports. This incorporates the Weber-Fechner law of the relation between human feeling and physical stimulus, and clearly shows the two extreme areas in which satisfactory achievement is possible. The dividing line encloses the area of insufficient report marks. In Switzerland the mark 6 is the best, the mark 4 is just sufficient and the mark 1 is the weakest one.

The sum of the ticks a student collects in the course of a semester is used to calculate the mean value h of ticks per task. It ranges from h = 0 to h = 3. If P is the mean of the product marks, the report mark Z is calculated as follows:

1. *h* is used to calculate W, the 'path mark':

   $W = 1 + 2.5 \log_2(h+1)$

2. P and W are used to calculate *z*:

   $$z = \frac{W + P}{2} + 0.3 \frac{(W-P)^2}{12-W-P} \quad \text{if } W \text{ and } P \text{ are not 6}$$

3. Finally z is rounded to the next whole or half number.

'Path' marks and product marks can, as the project showed, diverge widely. High-achieving students can do work which is good in comparison to their classmates, but weak in respect of their own capabilities. School would be doing them an injustice if it did not make this distinction, because it would not be furthering the children's talents, indeed would be allowing reserves of achievement to lie fallow. But weaker students are also done an injustice if their achievements are not given differential assessment. If teachers do not recognise that a child's achievement may be below the norm, but good with respect to the child's ability, they undermine the will to achieve and leave the child to cope alone with a diffuse feeling of inadequacy. Children thus lose the opportunity to assess their own achievements realistically and to improve their performance. Sooner or later they will refuse to work even to the level of which they would easily be capable.

Our non-linear, two-dimensional system of assessment was an attempt to do justice to individual achievement development without losing sight of supra-individual criteria. For several reasons it may appear unsatisfactory to compress the assessment into a single report mark. We do not see this as a final solution to marking problems, but as a significant improvement on the linear, one-dimensional system which gives insufficient stimulus for individual creativity and progress.